Price cap regulation in the postal sector: single vs multiple baskets[1]

Claire Borsenberger[*], Sébastien Bréville[†], Helmuth Cremer[‡], Philippe De Donder[§] and Denis Joram[**]

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[*] La Poste
[†] La Poste
[‡] IDEI, Toulouse School of Economics
[§] IDEI, Toulouse School of Economics
[**] La Poste

1 INTRODUCTION

Starting in the mid-1980's, many incumbents have become subject to price caps. Roughly speaking, a price cap defines an average price level not to be exceeded by the operator. The operator is otherwise free to adjust the relative prices of its different products. Price cap regulation is an extreme form of incentive regulation which provides powerful incentives for cost reduction. The system is also intended to reduce political interference in the setting of individual prices and the resulting price flexibility may also enable incumbent operators to become more business oriented. While their average price is capped by the regulator, they could adjust the prices of their different products to reflect costs, elasticity of demand, complementarities between segments, and competitive pressure.

In the postal sector, price cap schemes are by now widely used. However, in reality their design differs, often significantly, from the idealized policy just described. In particular, regulators very often are tempted to add additional constraints limiting price variations of so-called sub-baskets. This is justified by the need to offer special protection to some group of customers who for a variety of reasons may receive an extra weight in the regulator's objective. For instance, the regulator may want to restrict the increase in single-piece rates. In addition, it is sometimes argued that sub-baskets may restrict the incumbent's ability to engage in anti-competitive behavior (e.g. "squeezing" competitor's markups).

In this chapter, we examine the design of price cap schemes in the postal sector. In particular, we analyze whether or not it is appropriate to impose sub-baskets. First, we analyze a setting where some customers (or products) receive an extra weight in the regulator's objective and study to what extent this provides an argument for the imposition of extra constraints. The literature has shown that a global price cap can be used to decentralize Ramsey prices (that maximize unweighted surplus). We show that this result can be generalized in a variety of directions including welfare weights and the presence of competition. Second, we examine the role of demand uncertainty, which is certainly a major feature in today's postal markets. Sub-baskets introduce *ex ante* constraints on the pricing structure, which can prevent the operator from abusing market power in some states of nature. On the other hand, they reduce the possibility *ex post* to adapt relative prices to the realized structure of demand. We show that while the overall effect appears to be ambiguous, a number of arguments argue against the imposition of sub-baskets. In particular, we show that *ex ante* constraints tend to increase the overall price level.

The analysis thus leads to similar conclusions and argues in favor of a price-cap with a single basket. In other words**,** the constraint caps the (appropriately weighted) *average* price of all the regulated products. In either case, we restrict ourselves to the simplest possible model that can represent the main effects and tradeoffs involved. In particular, we assume for notational convenience that the operator sells only two products and we ignore dynamic issues (multi-period tradeoffs). The generality of our results will be discussed in detail below.

The remainder of this chapter is organized as follows. Section 2 studies the impact of welfare weights on Ramsey prices and on their decentralization through a price cap. Section 3 examines the effect of demand uncertainty. Finally, section 4 concludes.

2 REDISTRIBUTION AND WELFARE WEIGHTS

In regulation models (and more generally in the industrial economics literature), welfare is often measured by total surplus. This approach relies on a number of implicit or explicit assumptions (like the quasi linearity of preferences). Most significantly, it also assumes that the regulator is concerned with efficiency only. This simple setting is most useful to address many of the relevant issues. However, in reality, issues of redistribution, fairness or equity are omnipresent in the regulatory debate. In this section, we study how such a concern for redistribution affects the design of price-cap regulation. To do so, we assume that the regulator maximizes a weighted sum of individual utilities (surpluses). Specifically, the consumers of some type of product receive a larger weight. This relies on a very stylized view of the postal sector. There are two postal products (say single-piece and bulk mail) and two types of customers (say households and firms). Households consume single-piece mail while firms send bulk mail. For some reason, the welfare function puts a larger weight on households than on firms so that the regulator will be particularly concerned by the single piece rate. We examine if this concerns requires a specific regulation of each price or whether a price-cap specifying the average price level is sufficient.

2.1 The Regulator's Problem

There are two postal products, single-piece mail x , consumed by individuals of type h, and z, bulk mail, consumed by type f customers. Let $V_h(x)$ and $V_f(z)$ denote the consumers' utilities, while the demand functions are denoted $x(p_x)$ and $z(p_z)$. Marginal costs of both products are constant and given by c_x and c_z. In addition, the single operator has a fixed cost F . Assume that the regulator maximizes a weighted sum of the customers' utilities (putting a higher weight on type h individuals) subject to the operator breaking even

$$\max_{p_x, p_z, \lambda} (1+\beta)V_h[x(p_x)] + V_f[z(p_z)] +$$
$$(1+\lambda)[(p_x - c_x)x(p_x) + (p_z - c_z)z(p_z) - F], \tag{1}$$

where $\beta \geq 0$ represents the extra weight attached to type h individuals and λ the Lagrange multiplier of the operator's break-even constraint.

Differentiating (1) with respect to p_x and p_z and rearranging the first-order conditions, we obtain the following expressions for the optimal prices p_x^* and p_z^*

$$\frac{p_x^* - c_x}{p_x^*} = \frac{(\lambda^* - \beta)}{(1+\lambda^*)} \frac{1}{\varepsilon_x}, \tag{2}$$

$$\frac{p_z^* - c_z}{p_z^*} = \frac{\lambda^*}{(1+\lambda^*)} \frac{1}{\varepsilon_z}, \tag{3}$$

where ε_x and ε_z denote the absolute values of the demand elasticities.

These expressions reflect both efficiency and equity considerations. Like in the pure efficiency Ramsey pricing problem, we obtain inverse elasticity rules.[2] Equations (2) and (3) show how the prices are affected by the welfare weights. Not surprisingly, a higher level of β tends to reduce the price of the product consumed by households. Observe that when β is sufficiently large, it may be optimal to set p_x below marginal cost.

2.2Pricing of the operator under price cap regulation

In the previous subsection, we have assumed that the regulator has full control and directly sets all the operator's prices. We now show that this solution can be *decentralized* through a price-cap regulation that restricts solely the "average" level of *all* prices. In other words, a profit-maximizing operator submitted to the appropriate price cap constraint "spontaneously" sets the socially optimal levels of all prices. There is no need to use multiple baskets for which separate constraints are imposed. This property is well known for the standard Ramsey case where the regulator is concerned with efficiency only and does not use any welfare weights.[3] The appropriate price cap constraint states that the *weighted* average of all the prices does not exceed a certain threshold. Observe that the weights are *exogenous* from the operator's perspective. Specifically, the Ramsey solution is achieved when the weight attached to the price of a given commodity in the price cap formula corresponds to the socially *optimal* quantity of the commodity (*i.e.*, the quantity that solves the regulator's problem).[4] We shall now show that this result continues to apply in the case considered in the previous sub-section where welfare weights are used. The argument we present is inspired by Billette de Villemeur *et al.* (2002) who obtain similar results in a different context.

Assume that the postal operator is subject to the following price cap constraint,

$$\alpha_x p_x + \alpha_z p_z \leq \overline{p}, \tag{4}$$

where α_x and α_y are exogenous weights while $\overline{p}$ is a constant. A profit-maximizing operator then solves the following problem.

$$\begin{aligned} \max_{p_x, p_z, \mu} \; & (p_x - c_x)x(p_x) + (p_z - c_z)z(p_z) - F \\ & -\mu(\alpha_x p_x + \alpha_z p_z - \overline{p}). \end{aligned} \tag{5}$$

Differentiating (5) with respect to p_x and p_z and rearranging the first-order conditions yields the following expressions for the optimal prices $\tilde{p}_x$ and $\tilde{p}_z$:

$$\frac{\tilde{p}_x - c_x}{\tilde{p}_x} = \left(1 - \frac{a_x \tilde{\mu}}{x}\right)\frac{1}{\varepsilon_x} \tag{6}$$

$$\frac{\tilde{p}_z - c_z}{\tilde{p}_x} = \left(1 - \frac{a_z \tilde{\mu}}{z}\right)\frac{1}{\varepsilon_z} \tag{7}$$

Recall that these prices are optimal from the operator's perspective (solution to (5)). One can expect these prices $\tilde{p}_x$ and $\tilde{p}_z$ to differ in general from the socially optimal prices p_x^* and p_z^* determined by (2) and (3). However, $\tilde{p}_x$ and $\tilde{p}_z$ depend on the weights α_x and α_z of the commodities in the price cap formula. When these weights are set in an appropriate way, one can have $\tilde{p}_x = p_x^*$ and $\tilde{p}_z = p_z^*$. In other words, a profit-maximizing monopolistic operator will spontaneously choose the socially optimal prices provided of course that the weights are set at their appropriate levels.

To establish this result formally, define

$$\alpha_x = (1+\beta)x^*, \tag{8}$$

$$\alpha_z = z^*, \tag{9}$$

$$\overline{p} = \alpha_x p_x^* + \alpha_z p_z^*. \tag{10}$$

It is then sufficient to compare (2)–(3) and (6)–(7) to show that $\tilde{p}_x = p_x^*$, $\tilde{p}_z = p_z^*$, while $\tilde{\mu} = 1/\left(1+\lambda^*\right)$.[5]

When $\beta = 0$ we have a standard (pure efficiency) Ramsey pricing problem, which is decentralized by using the optimal quantities as weights in the price-cap formula. When good x receives a larger weight in the welfare function, the weight in the price cap formula has to be multiplied by the same factor (namely, $1+\beta$).

To implement this outcome directly the regulator has to know that optimal quantities. However, this solution can also be achieved in a more indirect way through an iterative procedure *à la* Vogelsang and Finsinger (1979).[6] The idea is to set the weight of the goods in period *t* respectively equal to $\alpha_x^t = (1+\beta)x^{t-1}$ and $\alpha_z^t = z^{t-1}$. In words, the weight in period t is equal to the quantity produced in the previous period times the welfare weight ($1+\beta$ for x and 1 for z).[7] This algorithm works provided that the managers of the operator take the weights in any given period as given. In other words, they are myopic in the sense that they do not take into account the impact of their choice and the future design of regulation.[8]
While we have established this result in the simplest possible setting, it reflects effectively a very general property. It depends neither on the separability of demand nor on the assumption that each individual only consumes a single product. The result also generalizes to the situation where the regulated firm is not a monopoly but competes with other operators. In either case, the appropriate weight of a product is given by the (partial) derivative of welfare with respect to its price (evaluated at the optimum solution). In the Ramsey case, the derivative of welfare with respect to p_x is equal to x^*.[9] With the welfare function defined in (1), the derivative is $(1+\beta)x^*$. With more general preferences and/or under competition, the expressions will be more complicated, but the basic rule determining the optimal weight remains the same. In all cases, the single price-cap constraint is sufficient to decentralize the (socially) optimal pricing policy.[10]

3 UNCERTAIN DEMAND

We now turn to the second aspect that may affect the design of price cap regulation, namely demand uncertainty. The notion of pricing flexibility of course becomes most relevant when future demand is uncertain at the moment when the price cap formula is determined. This argues for a single price cap formula with no sub-baskets, which allows the operator to adapt the pricing structure to the realized demand conditions. However, there might also be the danger that this flexibility allows too much market power to the operator, allowing it to extract excess profits in some states of nature. Consequently, one can expect that there is a tradeoff between costs and benefits of flexibility.

We represent this tradeoff in a simple model. Consequently, we cannot expect general results (such as a property that one solution *always* dominates the other one). However, the simple model brings out the different effects that are at work and illustrates the factors which are relevant to determine the appropriate degree of flexibility.

3.1 Model of Price Caps under Uncertainty

Consider an operator who sells two products, indexed by $k = 1,2$, whose demands, x_k, are given by

$$x_k = \theta_k - p_k, \tag{11}$$

where θ_k is a random variable, while p_k denotes the price. Suppose that $\theta_k \in \{\theta_L, \theta_H\}$, where

$\theta_L < \theta_H$, and where both realizations have the same probability of $1/2$.[11] In words, demand can be low or high in each of the market (which yields a total of 4 possible combinations). Observe that demand shocks only affect the intercept of the demand function and not its slope. Let $\overline{\theta} = E(\theta) = (\theta_L + \theta_H)/2$ denote the expected level of θ. Assume that θ_k's are distributed independently. Marginal production costs are constant and normalized to zero. There is a (joint) fixed cost, F, which is independent of demand uncertainty. Using the demand functions (11), one easily derives the following expression for consumer surplus, *CS*, total surplus, *TS*, and profits, π

$$CS = \sum_k \frac{(\theta_k - p_k)^2}{2}, \tag{12}$$

$$TS = \sum_k \frac{(\theta_k - p_k)^2}{2} + \pi, \tag{13}$$

$$\pi = \sum_k p_k(\theta_k - p_k) - F. \tag{14}$$

Maximizing (14) with respect to p_k yields the (profit-maximizing) monopoly price $p_k^M = \theta_k/2$. The corresponding level of total profits is given by

$$\pi^M = \left(\theta_1/2\right)^2 + \left(\theta_2/2\right)^2 - F. \tag{15}$$

Observe for future reference that the monopoly profit is positive in the worst state of nature ($\theta_1 = \theta_2 = \theta_L$) if and only if $2\left(\theta_L/2\right)^2 > F$.

We consider two methods of regulatory pricing. The first procedure, referred to as administered prices, is to set both prices *ex ante*, while the second one is to impose a price cap specifying the average price *ex ante*, while letting the operator choose the price structure *ex post.* In other words, the operator sets both prices once demand is known, but has to respect the price cap constraint (imposed *ex ante*). We can think about the first procedure as a price cap with sub-baskets, while the second one corresponds to a price cap with a single basket.

We shall now successively examine these two solutions. In either case, the regulatory policy is chosen *ex ante* and subject to the constraint that the operator's expected profits are zero, that is $E(\pi) = 0$. Consequently, (12)–(14) imply $E(TS) = E(CS)$ so that (expected) total surplus is measured by (expected) consumer surplus.

3.2Administered prices

The regulator maximizes expected surplus and sets both prices *ex ante* subject to the operator's (expected) break-even constraint $E(\pi) = 0$. With our symmetry assumptions, the solution to this problem is very simple. Demand and costs conditions for both products are identical *ex ante*. Consequently, it is plain that their prices should be equal and we have $p_1 = p_2 = p^*$. This price level is obtained by setting the expected value of (14) equal to zero and by taking the smallest root of this quadratic equation

$$2p^*(\overline{\theta} - p^*) = F. \tag{16}$$

Using equation (12)–(14) one can determine the corresponding levels of welfare $E(ST) = E(SC)$ which is given by[12]

$$W^{ap} = \left(\overline{\theta} - p^*\right)^2 + \frac{\left(\theta_2 - \theta_1\right)^2}{4}. \tag{17}$$

3.3Price cap

Suppose now that the regulator imposes a price cap defined by

$$\frac{p_1 + p_2}{2} \le \overline{p}, \tag{18}$$

where $\overline{p}$ is determined to ensure $E(\pi) = 0$. In this formula both products obtain the same weight, which is the optimal policy given the symmetry of the problem. The operator sets p_1 and p_2 *ex post* to maximize profits subject to (18). Depending on the realizations of demand, four different configurations can arise (two of which being equivalent). To determine the expected welfare achieved under the price cap regime, we have to determine the price set by the operator in the following three situations.

<u>Case 1:</u> $\theta_1 = \theta_L$ and $\theta_2 = \theta_L$

This is the most unfavorable state of nature, in which demand is low in both markets. It occurs with a probability of $1/4$. The operator then chooses

$$p_1 = p_2 = \min\left[\frac{\theta_L}{2}, \overline{p}\right]. \tag{19}$$

Given the symmetry of the demand realizations, the operator sets the same price in both markets. This price can either be equal to the monopoly price (under low demand), namely $\theta_L/2$, or set at the price ceiling $\overline{p}$ (whichever of these two prices is lower). In other words, it is possible that the price cap set *ex ante* by the regulator is effectively larger than the monopoly price under low demand. This will happen when the revenue under low demand ($2(\theta_L/2)^2$) is significantly lower than the fixed cost, so that large losses are incurred in this case.[13]

<u>Case 2:</u> $\theta_1 = \theta_H$ and $\theta_2 = \theta_H$

This is the most favorable case, for which demand is high in both markets (and occurs with probability $1/4$). Once again, it follows from the symmetry of the problem that the operator sets the same price in both markets. It follows immediately that this price is equal to the price cap level

$$p_1 = p_2 = \overline{p}, \tag{20}$$

as long as $\overline{p} < \theta_H/2$, which necessarily holds when $\overline{p}$ is set to ensure $E(\pi) = 0$.[14]

<u>Case 3:</u> $\theta_1 = \theta_L$ and $\theta_2 = \theta_H$ or $\theta_1 = \theta_H$ and $\theta_2 = \theta_L$

In words, demand is low for one good and high for the other. Given the symmetry of our problem, these two cases are equivalent and this configuration thus occurs with a probability of $1/2$. The operator now chooses different prices for the two products. Maximizing the operator's profit (14) subject to the price cap (18) yields

$$p_k = \overline{p} + \frac{\theta_k - \overline{\theta}}{2}. \tag{21}$$

Not surprisingly, the price is an increasing function of θ_k so that it will be larger in the market with high demand than in the market with low demand. Welfare in this case is given by

$$W^{LH} = \left(\bar{\theta} - \bar{p}\right)^2 + \frac{9\left(\theta_2 - \theta_1\right)^2}{64}. \tag{22}$$

Using (22) along with the welfare levels implied by the prices defined by (19) and (20) and weighting with the respective probabilities, one can then determine the level of expected welfare associated with the price cap policy, W^{pc}. As illustrated in the next section, this welfare level can then be compared with the level of welfare achieved in the administered prices regime, namely W^{ap} given by equation (17).

While these welfare comparisons are complex, expression (21) suggests that the increased flexibility the operator enjoys in the price cap regime may in fact also benefit the customers. Indeed, it shows that prices better reflect demand conditions than under the other policy; they are higher when demand is high and lower when demand is low and this is what an efficient pricing policy calls for. Furthermore, one obtains an unambiguous analytical result comparing the expected price level under the two policies. A simple argument shows that $\bar{p} \leq p^*$.[15] In other words, the larger flexibility allows break-even (coverage of fixed costs) with a smaller expected price level.

We now turn to the numerical illustration which shows that the (single basket) price cap policy can effectively yield a higher welfare while pointing out more clearly than the analytical expression how this result comes about.

3.4 Numerical example

Let $\theta_L = 5$, $\theta_H = 15$ and $F = 50$. Solving the break even constraint (16) yields $p^* = 5$ (as unique root). With this price, one easily calculates consumer surplus in the different states of nature: 0 for LL, 100 for HH and 50 for LH or HL. Consequently, expected surplus is given by $(0+100+2*50)/4 = 50$; this value can also be directly obtained from (17).

Price cap								
State of nature	p_1	p_2	x_1	x_2	CS_1	CS_2	π	TS
LL	2.5	2.5	2.5	2.5	3.1	3.1	-37.5	-31.3
HH	3.2	3.2	11.8	11.8	69.7	69.7	25.5	164.8
LH	0.7	5.7	4.3	9.3	9.3	43.3	6.0	58.5
Expected	3.0	3.0	7.0	7.0	31.3	31.3	0.0	62.6

Administered prices							
State of nature	$p^* = p_1 = p_2$	x_1	x_2	CS_1	CS_2	π	TS
LL	5.0	0.0	0.0	0.0	0.0	-50.0	-50.0
HH	5.0	10.0	10.0	50.0	50.0	50.0	150.0
LH	5.0	0.0	10.0	0.0	50.0	0.0	50.0
Expected	5.0	5.0	5.0	25.0	25.0	0.0	50.0

Table 1. Allocations in the different states of nature under the price cap (with $\bar{p} = 3.2$) and administered prices regime

Computations are more involved for the case of a price cap. Using the various expressions derived in previous subsection, one can determine first that $\bar{p} = 3.2$ yields $E(\pi) = 0$. This confirms the result that $\bar{p} < p^*$. With all prices above marginal cost, a lower expected price opens the door to the possibility of having a larger surplus under price cap. Observe that we have $\overline{p} > \theta_L/2 = 2.5$ (monopoly price with low demand LL). In words, when the demand is low in both markets, the operator will want to set both prices below the price cap (so that the constraint is effectively not binding).

With simple numerical calculations, one can then determine the level of consumer surplus in the different states of nature: 6.25 for LL, 139.3 for HH and 52.5 for LH and HL. Consequently, the expected surplus is given by $(6.25+139.3+2*52.5)/4 = 62.67$ which is effectively higher than the expected surplus achieved when prices are set *ex ante* (namely 50). The results of this numerical illustration are summarized in Table 1.

This finding confirms the conjecture suggested by the analytical results presented in the previous subsection, namely that the expected welfare (consumer surplus) may be higher under a price cap with a single basket than when sub baskets are imposed *ex ante.* As a matter of fact in the case presented in Table 1, the price cap regime gives a higher welfare (total surplus) in *all* states of nature. Furthermore, the example helps us understand why this result emerges. The single price cap has two advantages. First, the average price is smaller (recall that this is a general property). Second, the price can be adapted to reflect demand conditions. While this is always true, this may or may not be welfare enhancing in all possible states of nature. In our example, it turns out that the enhanced flexibility effectively benefits consumers in all states of nature; see Table 1. Observe that because $\overline{p} = 3.2 > \theta_L/2 = 2.5$, the expected price under the price cap regime (namely 3) is effectively below $\overline{p}$. This property will occur whenever the monopoly price in the least favorable state of nature for the operator is below the price cap.

4 CONCLUSION

We have examined the design of price cap schemes in the postal sector and have analyzed whether or not it is appropriate to impose “sub-baskets”. The imposition of such sub-baskets is sometimes justified by considerations of redistribution and the concern to “protect” certain groups of customers. Accordingly, we have first looked at a setting where some customers (or products) receive an extra weight in the regulator’s objective. We have shown that the socially optimal price structure can be *decentralized* through a price-cap regulation that restricts solely the “average” level of *all* prices. The appropriate price cap constraint states that the *weighted* average of all the prices does not exceed a certain threshold. While we have obtained this result in a simple setting, we have argued that the decentralization result, as well as the basic rule determining the optimal weight, remains the same with more general preferences and/or under competition. In all cases, the single price-cap constraint is sufficient to decentralize the (socially) optimal pricing policy. Formally, achieving this result requires that the regulator know the socially optimal quantities, but we also showed that the Vogelsang-Finsinger iterative approach can be used to converge to these optimal quantities, subject to the usual assumption of nonstrategic behavior on the part of the regulated firm.

Second, we have examined the role of demand uncertainty. In this context, sub-baskets introduce *ex ante* constraints on the pricing structure which can prevent the operator from abusing market power in some states of nature. However, they reduce the possibility *ex post* to adapt relative prices to the realized structure of demand. We have shown that while the overall effect appears to be ambiguous, a number of arguments plead against the imposition of sub-baskets. In particular, we have shown that *ex ante* constraints tend to increase the overall price level. Furthermore, they may prevent the price from being adapted to demand conditions.

The two sections thus lead to similar conclusions and argue in favor of a price-cap with a single basket. In other words**,** the constraint should cap the (appropriately weighted) *average* price of all regulated products.

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[2] The elasticity determines the "deadweight loss" of the implicit tax (markup over marginal cost) imposed on the good.

[3]See Laffont et Tirole (2000), page 67.

[4]When the optimal quantity is not known, the same result can be achieved through an iterative procedure under which the price in period *t* is weighted by the quantity sold in period *t*-1 ; see Vogelsang et Finsinger (1979).

[5]The weights defined by (8) and (9) generally do not add up to one. However, this is just a matter of normalization. To ensure that the sum of the weights equals one, it is sufficient to divide both sides of (4) by $(\alpha_x + \alpha_z)$.

[6] We thank John Panzar for bringing this property to our attention.

[7] The price cap $\bar{p}$ is set so that $\bar{p}^t = \alpha_x^t p_x^{t-1} + \alpha_z^t p_z^{t-1}$.

[8] The proof of this property follows directly from the proof of Proposition 1 in Vogelsang and Finsinger (1979). To accommodate the welfare weights one simply has to realize that the gradient of welfare (denoted grad *W(p)* in their proof) is now equal to the vector of quantities multiplied by the respective welfare weights. This follows from Roy's identity and the property that with quasi-linear preferences the marginal utility of income is equal to one.

[9] To see this, recall the discussion in footnote 7.

[10] It is plain that the result remains valid when there is a competitive fringe. Under imperfect competition the Ramsey problem is more complex because the other operator's equilibrium prices are affected by the regulated operator's prices. This problem is studied for instance by Ware and Winter (1986) under various assumption regarding the strategic interaction between operators (Stackelberg or Nash). One can easily show that their expressions can be decentralized by a price cap regulation with suitably designed weights. However, it is an open question at this point whether one can design an iterative procedure that converges to these weights.

[11]This specification is inspired by Laffont and Tirole (2000), p. 93–94. They introduce the demand functions (11) to study the impact of asymmetric information pertaining to the demand level. In their setting, θ_L applies to one product and θ_H to the other. [This is] Demand levels are observed by the operator but not by the regulator.

[12]Since we provide numerical illustrations below, we will refrain from reporting a full set of analytical expressions. Some of them are quite lengthy and their inspection is not very insightful.

[13]When the maximum profit with low demand in both markets is positive ($2(\theta_L/2)^2 > F$), we necessarily have $\bar{p} < \theta_L/2$. However, in the opposite case ($2(\theta_L/2)^2 < F$), we can have $\bar{p} > \theta_L/2$.

[14]A price cap exceeding the monopoly price in the most favorable demand condition ($\bar{p} > \theta_H/2$) would be a would be a completely empty constraint that is never binding.

[15]By definition p^* implies $E(\pi) = 0$. When $\bar{p} = p^*$, the operator has the option to set the price $\bar{p}$ in all states of nature (and realize a profit of zero). As our various pricing expressions show, this is of course not the optimal

policy. By adopting this optimal policy, the operator can then realize a strictly positive profit. Consequently, to achieve $E(\pi) = 0$, $\bar{p}$ can be set below p^*.